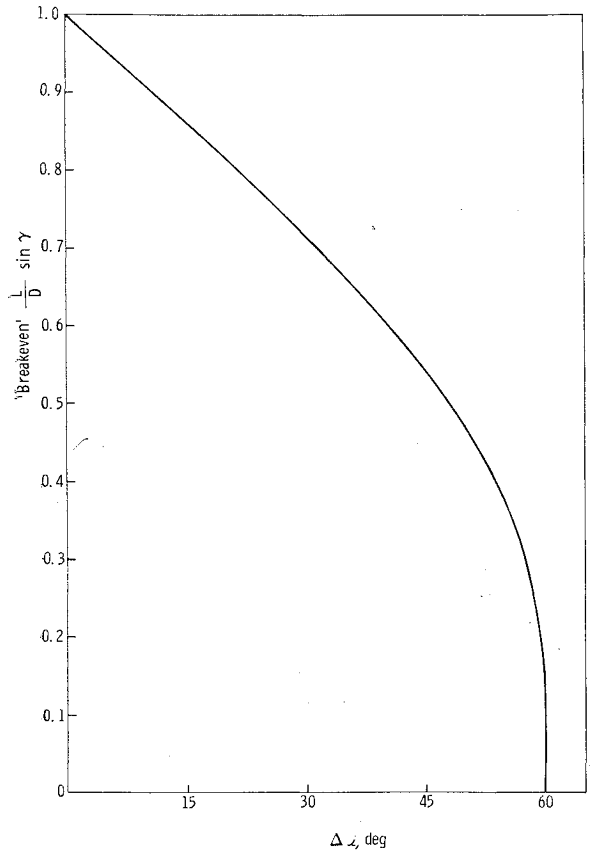

Fig. 2 "Breakeven" value of (L/D) sinγ vs Δi.

plane and the ballistic coefficient W/C_DA of the vehicle, as well as the characteristics of the atmosphere. The altitude and flight path angle at which a given velocity ratio and corresponding Δi are achieved will, of course, depend strongly on those details. Thus, in comparing aerodynamic plane change with extra-atmospheric plane change on the basis of rocket ΔV's required, limits can be defined for maneuvers of interest by calculating the minimum value of (L/D) sinγ required such that the velocity lost to drag during the aerodynamic maneuver is less than the ΔV required for extra-atmospheric plane change. The results of such a calculation for a single-impulse extra-atmospheric maneuver with the reference orbital velocity equal to V_E are shown in Fig. 2 for Δi up to 60°. The results for $\Delta i > 60°$ would be meaningless, since the extra-atmospheric maneuver ΔV would then equal or exceed V_E, which for the aerodynamic case would imply deceleration to zero or negative speed. The ΔV required for extra-atmospheric maneuvers can be reduced for Δi greater than about 30° by employing three-impulse maneuvers that require maneuvering times of more than one orbital period, as discussed in, e.g., Ref. 2. Even so, the "breakeven" value of (L/D) sinγ for Δi between 30° and 90° never exceeds 0.89, this number corresponding to the limiting case of a 90° extra-atmospheric plane change requiring infinite time. It should also be noted that the "breakeven" value of (L/D) sinγ will increase as the orbital altitude increases, since extra-atmospheric plane change can then be executed at lower velocity. The "breakeven" values of (L/D) sinγ discussed herein are of course only lower bounds since, in general, the ΔV needed to re-enter orbit is greater than the velocity decrement due to drag because of the necessity of changing both altitude and flight path angle. In addition, of course, is the rocket ΔV required to descend initially to the atmosphere unless entry is made on a shallow "lob" trajectory directly from ground launch rather than from a parking orbit.

In light of these remarks, the primary conclusion of Ref. 1 should be rephrased. It was stated therein that plane change by means of an aerodynamic skip maneuver will yield a ΔV saving as compared to an extra-atmospheric maneuver whenever the vehicle L/D is 1.0 or greater. This should be corrected to say instead that a ΔV saving may result, for any class of aerodynamic maneuver in which (L/D) sinγ is constant and θ remains small, if the value of (L/D) sinγ is equal to or greater than the "breakeven" value as discussed in this note and which cannot exceed 1.0 for $0 \leq \Delta i \leq 90°$. Whether or not a saving will actually occur depends upon the details of the maneuvers.

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Similarity Rule Estimation Methods for Cones

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Nomenclature

C_p = pressure coefficient = $2(p_2/p_1 - 1)/(\gamma M_1^2)$
M = Mach number
p = pressure, psia
T = temperature, °K
U = velocity, fps
β = $(M_1^2 - 1)^{1/2}$
γ = ratio of specific heats
θ = semiapex cone angle

Subscripts

1 = freestream
2 = cone surface

QUITE frequently, in electronic computer programs, equations are needed to predict the Taylor-Maccoll cone values of temperature, velocity, Mach number, and pressure. A set of such equations are presented in this note for an inviscid undissociated, supersonic flow around a cone. These equations are valid for $\gamma = 1.4$.

Using the hypersonic similarity parameter $M_1 \sin\theta$, the following equations were obtained for the cone surface to freestream temperature ratio. For $0 \leq M_1 \sin\theta \leq 1.0$,

$$T_2/T_1 = 1.0 + 0.35(M_1 \sin\theta)^{1.5} \tag{1a}$$

and for $M_1 \sin\theta \geq 1.0$,

$$T_2/T_1 = [1 + \exp(-1 - 1.52M_1 \sin\theta)][1 + (M_1 \sin\theta/2)^2] \tag{1b}$$

Data obtained from the cone tables of Sims[1] show little scatter when compared with Eq. (1) in Fig. 1.

The cone velocity was correlated by the following equation:

$$U_2/U_1 = \cos\theta[1 - \sin\theta/M_1]^{1/2} \tag{2}$$

In Fig. 2 the maximum error (about $4\frac{1}{2}\%$) occurred on the 30° cone near the shock detachment Mach number.

The Mach number ratio can be obtained by dividing Eq. (2) by the square root of Eq. (1). This is done in Fig. 3 and compared with the exact Taylor-Maccoll data of Sims.[1] Again, the maximum error is seen to occur near the shock detachment points.

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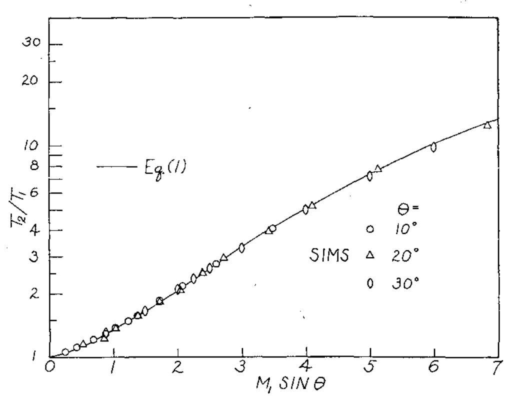

Fig. 1 Cone temperature ratio.

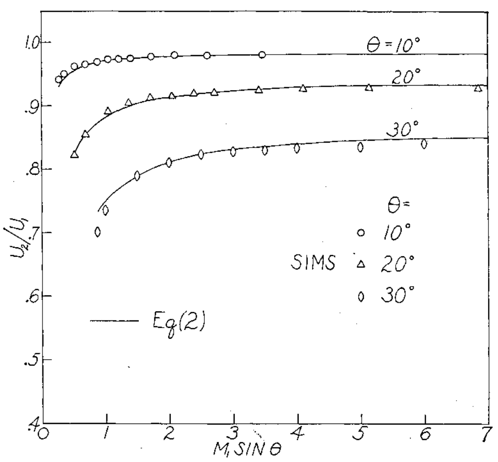

Fig. 2 Cone velocity ratio.

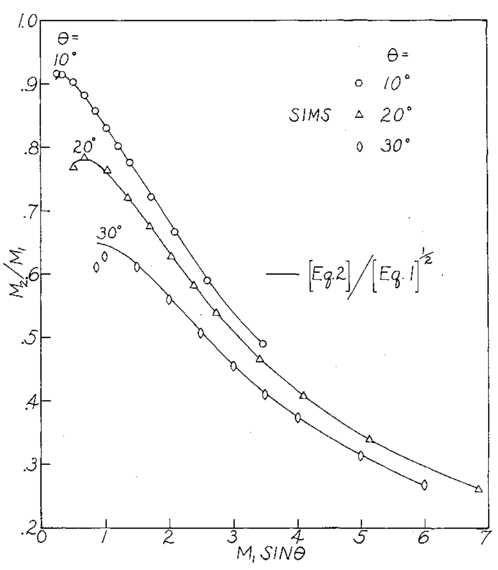

Fig. 3 Cone Mach number ratio.

Previously, Linnell and Bailey,[2] using the similarity concept, obtained the following expression for the pressure coefficient on a cone:

$$C_p = (4 \sin^2\theta)(2.5 + 8\beta \sin\theta)/(1 + 16\beta \sin\theta) \qquad (3)$$

Eq. (3) can be used to obtain the pressure on a cone surface, and this pressure in turn can be used with Eq. (1) and the equation of state to obtain the density on the cone surface.

The accuracy of Eq. (1) and Eq. (2) are questionable above a Mach number of 10, due to the dissociation effects. However, a comparison of Eq. (3) with the results of Romig[3] indicates that Eq. (3) is not affected to any large degree by dissociation.

References

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Effects of Nitrogen, Excess Hydrogen, and Water Additions on Hydrogen-Air Flames

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KUEHL[1] has measured the effect on the burning velocity of adding nitrogen, excess hydrogen, and steam to a premixed stoichiometric hydrogen–air flame at $\frac{1}{4}$ atm pressure and an initial gas temperature of 800°F (720°K). Whereas addition of nitrogen caused a reduction in burning velocity proportional to the nitrogen added, the addition of excess hydrogen produced increases in the burning velocity. Water caused only about one-third of the reduction produced by an equivalent volume of nitrogen. On replacing the nitrogen in the air by water vapor on a mole-for-mole basis, the burning velocity increased (from around 700 cm-sec^{-1} initially) by about 6 cm-sec^{-1}/mole % water in the over-all mixture.

One approach to the effect of inert additives is to assume that the additive acts as a heat sink, thus reducing the flame temperature, and hence the burning velocity. On this basis alone, it would be expected that all three additives would reduce the burning velocity, whereas heat capacity effects suggest that, mole for mole, water should be more efficient than nitrogen in this way. Assuming the nitrogen to behave in a "normal" fashion, the behavior of both the excess hydrogen and the water vapor is therefore anomalous.

In the case of the *water* additive, Kuehl suggests that the increase in burning velocity when nitrogen is replaced by water may be due to the ability of the unburned gas containing water to absorb radiation from the flame. On the other hand, in a recent note, Levy[2] has drawn attention to a possible chemical kinetic interpretation of the phenomenon. According to this, by analogy with slow reaction studies, the reaction rate in the flame may be increased by such steps as

$$H_2O + HO_2 = OH + H_2O_2 \qquad (1)$$

$$H_2O + H + O_2 = OH + H_2O_2 \qquad (2)$$

Arising from our work on the hydrogen-oxygen flame system, we would like to enlarge on this theme.

First, a slow-burning, hydrogen-rich, hydrogen-oxygen-nitrogen flame at atmospheric pressure has been studied experimentally.[3] The flame temperature was 1072°K and the burning velocity 9.2 cm-sec^{-1}. (Gas flows measured at 18°C and 760 mm.) The initial gas temperature was 336°K. Analysis of the measured temperature profile gave a maximum heat release rate of 20 (±2) cal-cm^{-3}-sec^{-1} at 900°K, whereas the maximum hydrogen atom concentration, measured by studying the rate of the $H + D_2$ exchange reaction, was about 2.5×10^{-8} mole-cm^{-3} at this temperature. Furthermore, at 900°K, $(O_2) \approx 1$,1 $(H_2) \approx 11$, and $(H_2O) \approx 8$ mole %.

Second, the widely accepted basic mechanism used to interpret the explosion limit and slow reaction behavior of

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